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TECHNICAL REPORT RG-75-48

DIGITAL AUTOPILOT SAMPLE RATE SELECTION BASED ON CONTROL SYSTEM REQUIREMENTS

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1 May 1975

U.S. ARMY MISSILE COMMAND

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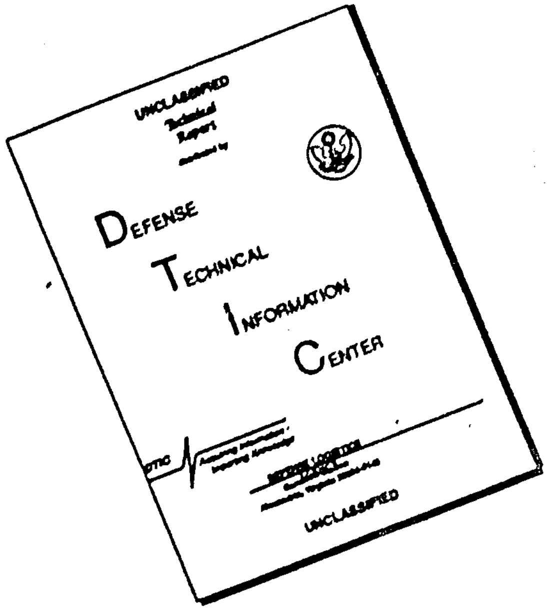

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: RG-75-48
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): DIGITAL AUTOPILOT SAMPLE RATE SELECTION BASED ON CONTROL SYSTEM REQUIREMENTS
5. TYPE OF REPORT & PERIOD COVERED: Technical Report
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Edward E. Herbert
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Guidance and Control Directorate, US Army Missile Research, Development and Engineering Laboratory, US Army Missile Command, Redstone Arsenal, Alabama 35801
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: AMCMSC 2300.23.5465.1.31
11. CONTROLLING OFFICE NAME AND ADDRESS:
12. REPORT DATE: 1 May 1975
13. NUMBER OF PAGES: 12 p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
    - Stability compensation
    - Missile autopilot
    - Microprocessor
20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

The purpose of this paper is to discuss some considerations in the selection of a sample rate for the digital autopilot. The missile autopilot design which incorporates a digital computer or microprocessor(s) to implement the stability compensation equations requires considerations over and above those made for analog computer autopilots. Some of these considerations are: the selection of sampling frequency, the A/D converter wordlength, the phase lag contributed by the zero order data hold (i.e., the A/D converter), the transport lag in the digital computer, and the effects of sampling high frequency sensor and structural (bending) noise (foldover or "aliasing").

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INTRODUCTION

Missile autopilot design which incorporates a digital computer or microprocessor(s) to implement the stability compensation equations requires considerations over and above those made for analog computer autopilots. Some of these considerations are: the selection of sampling frequency, the A/D converter wordlength, the phase lag contributed by the zero order data hold (i.e., the D/A converter), the transport lag in the digital computer, and the effects of sampling high frequency sensor and structural (bending) noise (foldover or "aliasing"). The purpose of this paper is to discuss some considerations in the selection of a sample rate for the digital autopilot.

In the following discussion a distinction is made between guidance and control. Guidance is assumed to be that aspect of the missile which determines the required flight path of the missile which causes target intercept. Control is assumed to be that portion which provides satisfactory missile behavior about that flight path.

ANALOG VERSUS DIGITAL CONTROL COMPENSATION (AUTOPILOT)

The analog control computer processes the error sensor outputs by comparing the measured response to the required or commanded response, and alters the error signal to provide correction commands to give desired response without oscillation.

The digital control computer also makes comparisons of desired versus measured response to determine the correct response command. However, the error determination is corrupted by noninfinite resolution A/D converters. The real world A/D converter not only gives quantization noise but often generates bit noise which further corrupts the error signals. At the opposite end of the computer, an additional problem is encountered. The digital computer can only determine the error signal once every sample; therefore, the output is not smooth but is constant (with a zero order hold) between samples. This has the effect of adding a delay to the system, and a delay is almost always destabilizing. Thirdly, the computer requires time to process the input signals in order to generate the output command. That time required is a second transport lag that, in general, is destabilizing.

If a satisfactory analog control autopilot has been formulated and tested, and then a digital autopilot is proposed, a common approach is to emulate the analog autopilot. This is especially common in situations which the plant (missile plus actuator, etc.) is not totally known because of lost aerodynamic data, actuator data, or whatever. This is the cause of much concern by the digital autopilot designer because in this case he does not really know just how much gain and phase margins existed in the first place. Thus he is forced into a situation where he tries to maintain the same gain and phase shift through the digital computer as was specified for the analog control computer. If he succeeds perfectly in gain and phase relationships, he still is burdened with the noise introduced by the sampler (A-D converter). In many missile systems, additional noise injected into the control loop has extremely detrimental effects on the actuation system, usually in the form of much greater hydraulic or electric power requirements. Thus the control engineer is faced with the requirement to emulate an analog system (which cannot be perfectly accomplished) without utilizing the advantages of flexibility and versatility offered by the digital autopilot. Flexibility allows the compensation to be changed almost constantly during flight to provide optimum stability margins rather than just one or two "bands" which are a compromise between the operating extremes expected. Versatility enables the system designer to incorporate options not readily available to the analog designer, such as a wide variety of open loop maneuvers, and to change the guidance law for different portions of flight.

DIGITAL CONTROL SYSTEM DESIGN

In order to utilize the analog control engineer to the maximum extent in the design of digital control systems, one technique often used is to develop an analog control compensator and then use the "Rader and Gold" method (Ref. 1) of converting the compensation to a digital compensator. This method preserves the gain and phase characteristics of the analog compensator to a frequency up to approximately one-third the sample frequency. It consists of prewarping the break frequencies and then making the following substitution for s

$$s = \frac{z-1}{z+1}.$$

The prewarping consists of altering the break frequency (a) by the following equation $a' = \tan \frac{aT}{2}$ where T = sample period. This design method works reasonably well. The control engineer frequently selects a sample frequency ten times the frequency of interest in order to maintain the gain and phase margins determined in his analog design. This method however, does not account for the transport lag of the digital computer nor the zero order hold phase lag unless approximations of these terms had been included in the analog system design. By not including these terms, the phase margins suffer by the following amounts. The zero order hold phase lag is

$$\phi_{LAG(ZOH)} = \frac{180 f_c}{f_s} \quad \text{(degrees)}$$

where f_c is the critical frequency (usually rigid body crossover frequency) and f_s is the sample frequency.

Thus if the design had been selected where there was a 30° phase margin at the 0db crossover frequency, the digital version would have only 12° phase margin assuming the sample rate 10 times crossover frequency had been selected. This phase margin is unacceptably small, as it means greater than a 14 dB resonant peak in the closed loop frequency response.

In addition, the computational delay in the digital airborne computer also provides a phase lag of the following amount

$$\phi_{LAG(DELAY)} = 360 f_c \times t_d \quad \text{(degrees)}$$

where t_d = delay in seconds

In practice, the digital airborne computer is sufficiently loaded as to require between 50% and 80% of its sample period to sample the input error sensors and compute the required output commands. Thus the delay can be found to be within the following range

$$360^* \frac{0.5}{f_s} {}^*f_c < \phi_{\text{delay (degrees)}} < 360^* \frac{0.8}{f_s} {}^*f_c$$

If the sample frequency had been selected to be ten times crossover frequency, then the delay phase lag has the range

$$18^\circ < \phi_{LAG(DELAY)} < 28.8^\circ.$$

The total phase lag for this typical case, with both zero order hold and computational delay, is between 36° and 46.8° which can give a negative phase margin in the system if these effects had not been included in the original analog design. To have included them in the original analog design would have required the use of Pade' approximations (Ref. 2) which increase the order of the system by the number of terms chosen in the approximation.

A more direct approach to digital autopilot design is through the use of Z-transforms. This technique involves no approximations of transport lags or zero order holds. There are several Z-transform computer programs (Ref. 3) which utilize the theory of residues in order to avoid series truncations to find Z-transforms.

These programs accept the Laplace transform definition of the plant and computational delay and give a ratio of polynominals in Z-domain, for which a suitable compensation is to be determined. This technique does not tell the designer what sample rate to select, but by choosing different sample rates and compensating the resultant equations, the designer can determine which sample rate best allows him to meet stability criteria with the minimum penalty (compensator order and amount of "lead" required). It has been shown that with missiles having lightly damped body bending roles, a high sample rate is most beneficial (Ref. 4). This is true because the phase lags of the computational delay and zero order hold are still present and must be accounted for by sufficient phase lead in the compensation. These items have not disappeared but have become more directly assessable by the Z-transform technique.

In general, analog control engineers are reluctant to use the Z-transform for the following reasons. First, a Z-transform program is not usually immediately available, or if it is available, there is no confidence in it because of the lack of experience. Secondly it requires a different criteria for stability in that the roots of the open loop transfer function must have magnitude less than unity (quadratic roots must be in polar form) and stability margins in the Z-domain are somewhat more difficult to assess, i.e., lines of constant damping in the unit circle are distorted.

In order to get around these objectives and to let the analog control engineer use continuous system analysis and design tools, a transformation has been made available which maps the Z-domain unit circle into a left half plane of a new variable W. The change of variables is

$$Z = \frac{1 + W}{1 - W}$$

and is a by-product of some Z-transform programs (Ref. 3). By the use of this transformation, the compensation designer can treat the open loop transfer function in W-domain as if it were in the S-domain, and all programs for root locus, Bode plots, Nichols charts, Nyquist plots, etc. for continuous systems are useable. Again, this does not reduce the problems associated with digital control autopilots but merely makes them more addressable.

A number of digital autopilot analysts have attempted to utilize sumulation techniques in transforming the analog design to a digital design. Others have tried matching the impulse response of the analog compensator by a difference equation having the same impulse response. It has been noted that those two methods fail to account for the computer delay and output hold, but also, they do not match the zero (numerator) frequency response of the desired compensator. (See appendix A). In general, simulation techniques should be avoided in sampled-data autopilot design.

BODY BENDING

So far only rigid body crossover frequencies and the phase lag of the digital system at these frequencies have been discussed. The body bending of missiles cannot be dismissed because quite often the actuation (controller) technique is sensitive to signals of these frequencies. Blowdown type actuators will require more reserve if the actuator encounters the body bending frequency. Hydraulic actuators may require greater pump capacity or accumulator reserve if these frequencies exist for any length of time. In a relatively large percent of the cases, the amplitude of the missile body bending (mode 1) is sufficiently great to cause the loop gain to be greater than 0 dB, therefore potentially unstable. The designer cannot always reduce this amplitude sufficiently, either because of its proximity to rigid body crossover frequency or because its range is too broad for a notch filter. Therefore, this mode is "phase stabilized" by making the phase at that frequency range sufficiently far from 180° that instability cannot occur. (See Figure 1). In analog systems a "lead" type compensator provides just enough phase lead to give the required phase margin (there is no gain margin). Should an attempt to "digitize" this analog design be made without accounting for the computational and hold lags, the phase margin would be lost with tragic results. For example, if mode 1 body bending had a frequency of 5 times rigid body crossover frequency, then the phase lag due to the delay and hold would be

$$180^\circ < \phi_{\text{delay}} < 234^\circ$$

where sample frequency is 10 times rigid body crossover frequency. For that reason, many analog control engineers who are attempting to phase-stabilize mode 1 body bending will select a sample rate of 10-15 times the bending frequency, thereby assuring that the delays of 20° to 30° can be relatively easily compensated for in in the compensation design. The same thing must be done in the direct Z or W-domain, but in a more direct fashion. In an analytical sense, the designer can provide an extremely large amount of lead, including multiple pure zeroes, but practical experience shows that this kind of design would greatly accentuate any sensor and A-D converter noise would quickly deplete the actuation system. By selecting high sample frequency to minimize the amount of lead compensation required to account for the delay, the autopilot designer suddenly realizes that he has chosen a sample which operates at 10-15 times mode 1 body bending frequency, which is perhaps 50-75 times rigid body crossover frequency - and this may be 100 to 150 times the missile rigid body airframe natural frequency.

These high sample frequencies introduce additional penalties. It has been shown (Ref. 1, p. 102) that the high sample rates generate new problems such as greater required minimum A/D converter wordlength (difficult to obtain at high sample rates) and internal computer wordlength or use of double precision arithmetic which increases computational delay.

Signal foldover, also called aliasing, has been given considerable attention by authors of digital autopilot theory papers (Ref. 5). In light of the high sample rate selection for phase considerations, signal foldover of rigid body and mode 1 bending is not a problem. Sensor noise near the sample frequency will probably be of low amplitude and folded signals should then be of little consequence. However, for safety margin, the compensation should include a second order lag filter with a break frequency just high enough to have little effect on the phase lag at the frequency of interest, which is rigid body crossover of mode 1 bending. Communication type filters such as Butterworth, Chebycheff, etc., cannot be used because of the large phase lags associated with those filters at low frequency compared to their stopbands.

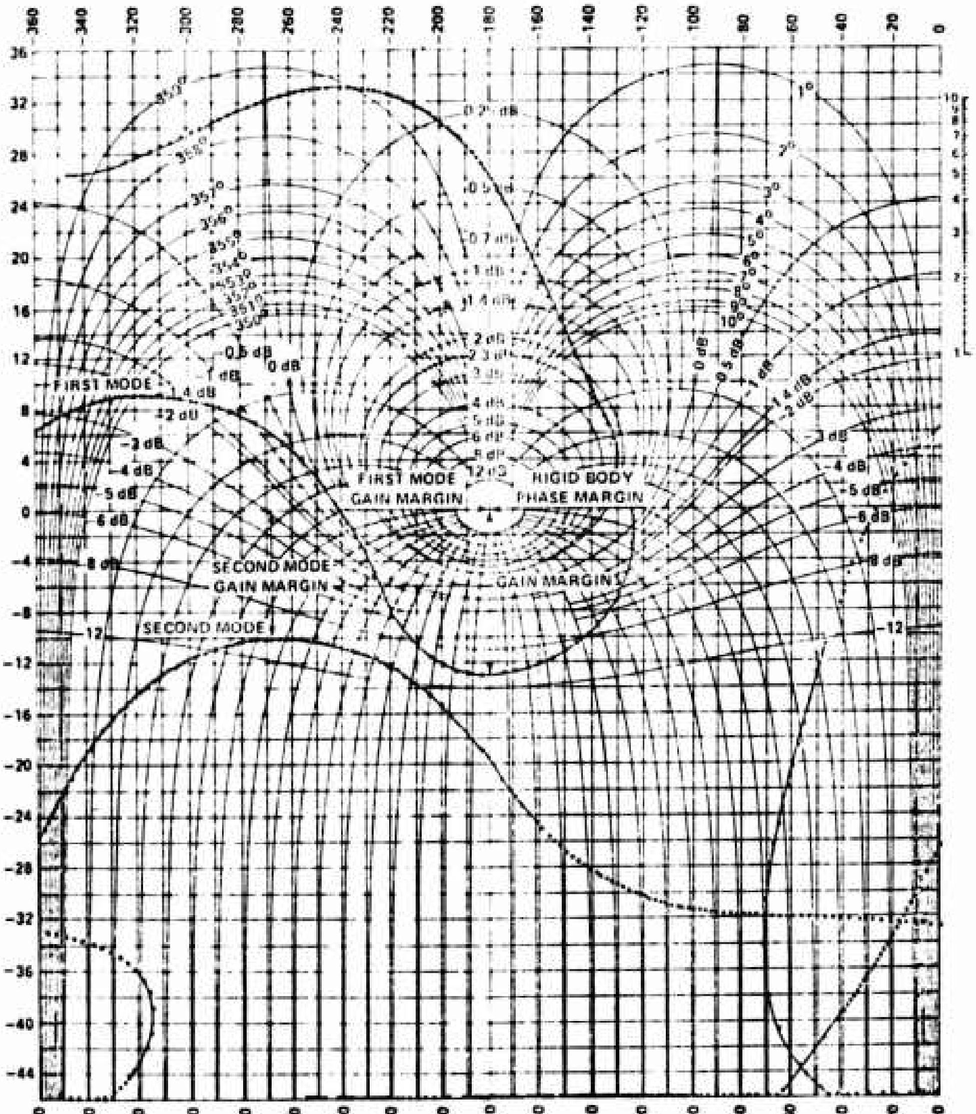


Figure 1. Nichols chart for sampled-data autopilot.

OPTIMAL CONTROLLERS

Optimal control theory has been applied to digital autopilots (Ref. 6) but, in general, this technique does not offer some of the advantages which classical theory provides. First, optimal control theory assumes that the system is a regulator and that the designer is attempting to zero all states with some minimization of control or state action. It also assumes infinite time to accomplish this result. The problem is presented in this manner to keep it mathematically tractable. Unfortunately, the feedback matrix coefficients are strongly dependent on knowledge of the plant (the A matrix) and feedback in the classical sense to compensate for uncertainties in the plant parameters is not provided. All system states must be fed back in the optimal controller. Unmeasureable states must be synthesized by state observers or estimators, which further increase the complexity of the total digital control autopilot. Although optimal control theory does not provide any quantum jump in stability margins over classical control theory, it does provide for controlling several variables, which is an improvement over single-input single-output systems.

Stochastic optimal control theory, in the form of Kalman filter theory, has provided results to estimate plant states from signals corrupted by noise, which has proven successful in the SAM-D autopilot. Digital autopilot design should certainly consider the Kalman filter, although it falls more in the domain of guidance than control. It should be considered then in the decision of digital computer sizing and memory as it offers benefits unavailable in most analog airborne computers. The guidance portion of the guidance and control of missiles is relatively low frequency, on the order of a tenth of the rigid body control frequencies. Consequently, the guidance equation is solved at a much slower rate than control, and only a portion of the guidance equations are solved in each control sample period. It may take 5 to 10 control sample periods to output a guidance command. This technique offers a problem for the control engineer, in that the entire guidance and control system is now a multiloop, multirate system. In the Z-domain compensation design this problem is frequently not solvable, as block diagram reduction for nonerror sampled systems is not possible (Ref 7). The autopilot designer must then assume that loop interactions do not occur.

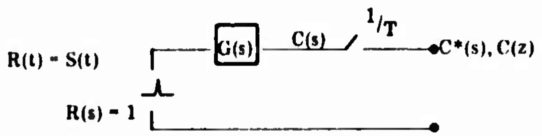
CONCLUSION

This report has discussed some of the considerations in the design of digital autopilots for missiles. A major aspect of the design is the selection of the sampling rate, since this factor dictates A-D converter rates, wordlength, and the computer speed requirement. Because the computational delay and the zero order hold impart phase lag inversely proportional to sampling frequency, the designer must select a sample rate 10 to 15 times mode 1 body bending frequency if it is phase stabilized. If bending amplitude is sufficiently low, a sample rate selection of 10 to 15 rigid body crossover frequency is sufficient to assure adequate rigid body phase margin. The digital computer provides the ability to adjust the compensation throughout the flight to obtain maximum vehicle response within stability constraints, while most analog autopilots have a single configuration which is required to give reasonable stability margin over a wide range of flight conditions. The guidance aspect of missile guidance and control can most benefit from the modern techniques of Kalman filteri.g and optimal response.

APPENDIX A
IMPULSE INVARIANT TRANSFORMATIONS FOR DIGITAL COMPENSATION

The response of an initially relaxed system of differential equations to a unit impulse is equal to the inverse Laplace transform of the transfer function. Thus the transfer function is really the transform of the impulse response.

If the impulse response of the system is sampled, the Z-transform of the response gives the output at the sample instants. The Z-transform is referred to as an impulse-invariant transformation because of that characteristic. The following diagram depicts the situation.



where $G(z) = Z\ (G^*(s))$

If G(s) is an integrator $\frac{1}{s}$, the following output results:

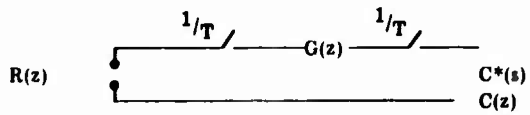
$$C(z) = \frac{z}{z-1} = Z\ (G^*(s)).$$

The inverse of this transform gives the time function of the output. In this case,

$$C^*(t) = 1,$$

which means an output of unit amplitude at each sample instant.

Control Engineers seek compensation equations which are transfer functions whose impulse response causes the system to meet error and stability requirements. Some engineers have attempted to "digitize" an analog compensator by implementing the Z-transform by a set of difference equations. They feel that since the Z-transform is an impulse invariant transformation, this technique should work. However, that method is incorrect for the following reason. The replacement of the analog transfer function by a difference equation set is shown in the following diagram.



The situation is dramatically different from the preceeding diagram by the addition of the samples in front of the compensator. As is pointed out in many tests (1),

$$Z\ (\ R(s)G(s)\ ) = RG(z) \neq R(z)G(z).$$

(1) "Introduction to Continuous and Digital Control Systems." Roberts Saucedo and Earle Schiring, MacMillian, 1968, p. 231.

In the first figure, the output was

$$C(z) = RG(z);$$

in the second figure

$$C(z) = R(z)G(z).$$

A few trials with various inputs R(Z) will demonstrate the inequality. This problem is not dependent on sample rate, but is always true.

The same sort of matching technique described above has been tried by engineers using the State Space methods of Modern Control Theory. In their method the analog compensation equations are rewritten as first order differential equations called State Equations. Then they attempt to find a set of first order difference equations whose outputs at the sample instants is equal to the analog state equations excited by a unit impulse input vector. That is equivalent to the first described classical approach and equally erroneous results will be obtained.

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